4.8 Fermi Surfaces

At very low temperatures electrons fill the energy bands of solids up to a energy called the Fermi energy E_F, and the bands are empty for energies that exceed E_F. In three-dimensional k space the set of values of k_x, k_y and k_z which satisfy the equation $\hbar^2 (k_x^2 + k_y^2 + k_z^2)/2m = E_F$, form a surface called the Fermi Surface. All k_x, k_y, k_z energy states that lie below this surface are full, and the states above the surface are empty. The Fermi surface encloses all the electrons in the conduction band that carry electric current.

An intrinsic semiconductor is one with a full valence band and an empty conduction band at absolute zero temperature. At ambient temperatures some electrons are thermally excited to the bottom of the conduction band, and an equal number of empty sites or holes are left behind near the top of the valence band. This means that only a small percentage of the Brillouin zone contains electrons in the conduction band, and the number of holes in the valence band is correspondingly small. In a one-dimensional representation this reflects the electron and hole occupancies depicted in Fig. 4.11.

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If the conduction band is minimum at the Γ point in the center of the Brillouin zone, as is the case with GaAs, then it will be very close to a sphere since the symmetry is cubic, and to a good approximation we can assume a quadratic dependence of the energy on the wave vector k, Therefore the Fermi surface in k space is a small sphere given by the standard equation for a sphere

$$E_F = E_g + \frac{\hbar^2}{2m_e}(k_x^2 + k_y^2 + k_z^2)_F$$

$$= E_g + \frac{\hbar^2}{2m_e} k^2 F$$

where E_F is the Fermi energy, and the electron mass, m_e relative to the free-electron mass for various direct-gap semiconductors. The situation for the conduction electron Fermi surface is more complicated for the indirect gap semiconductors. The corresponding six ellipsoidal energy surfaces are sketched in Fig. 4.13 (*b*). The logitudinal and transverse effective masses, m_L and M_T respectively the following values

$$\frac{M_L}{M_o} = 0.92 \quad \frac{M_T}{M_o} = 0.19 \text{ for Si}$$

$$\frac{M_L}{M_o} = 7.25 \quad \frac{M_T}{M_o} = 0.21 \text{ for GaP}$$

for these two indirect-gap semiconductors.

Germanium has its band minimum at points L of the Brillouin zone sketehed in Fig. 4.7, and its Fermi surface is the set of ellipsoids centred at the L points with their axis along Λ or (III) directions, as shown in Fig. 4.13 (*a*). The longitudinal and transverse effective masses for these ellipsoids in Ge are $m_L/m_o = 1.58$ and $m_T/m_o = 0.081$ respectively.

4.9 Fermi Energy

The electron energy spectrum in a solid consists of bands. These bands show the allowed electron energy states. According to the Pauli exclusion principle, we know that the electrons fill the atomic orbitals with lower energies first, similarly the electrons in a crystal fill the lower energy bands first.

A solid having m energy levels and n electrons at equilibrium. Usually these numbers are extremely large and the number m of allowed energy levels (taking into account the spin degeneracy) in a solid in much larger than the number n of electrons ($m>>n$).

For example an iron metal with a volume of 1 cm^3 will have approximately 10^{22} atoms and 10^{24} electrons. At equilibrium, when no electron is in an excited state (at the absolute zero temperature 0 K), the lowest n energy levels will be copied by electrons and the next remaining m-n energy levels remain empty.

The highest occupied state is inside a band, the energy of this state is called the **Fermi level** and is denoted by E_F. That band is therefore only partially filled. This is illustrated in Fig. 4.14 (*b*) for metals. In case of semiconductors all bands are either full or empty.

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The Fermi level thus lies between the highest energy fully filled band (valence band) and the lowest energy empty band (conduction band) as shown in Fig. 4.14 (a). The energy gap between the valence band and the conduction band is called the band gap and is represent by E_g.

The location of the Fermi level relative to the allowed energy bands is important in determining the electrical properties of a solid. Metals have a partially filled free electron band, since the Fermi level lies inside this band, which makes metals good electrical conductors as the higher energy electrons can readily be excited into an empty energy state and contribute to electrical conduction. At 0 K most semiconductors have completely filled or completely empty, electron bands which means that the Fermi energy lies inside a forbidden energy gap and hence they are poor electrical conductors at low temperatures. Same can be applied about insulators.

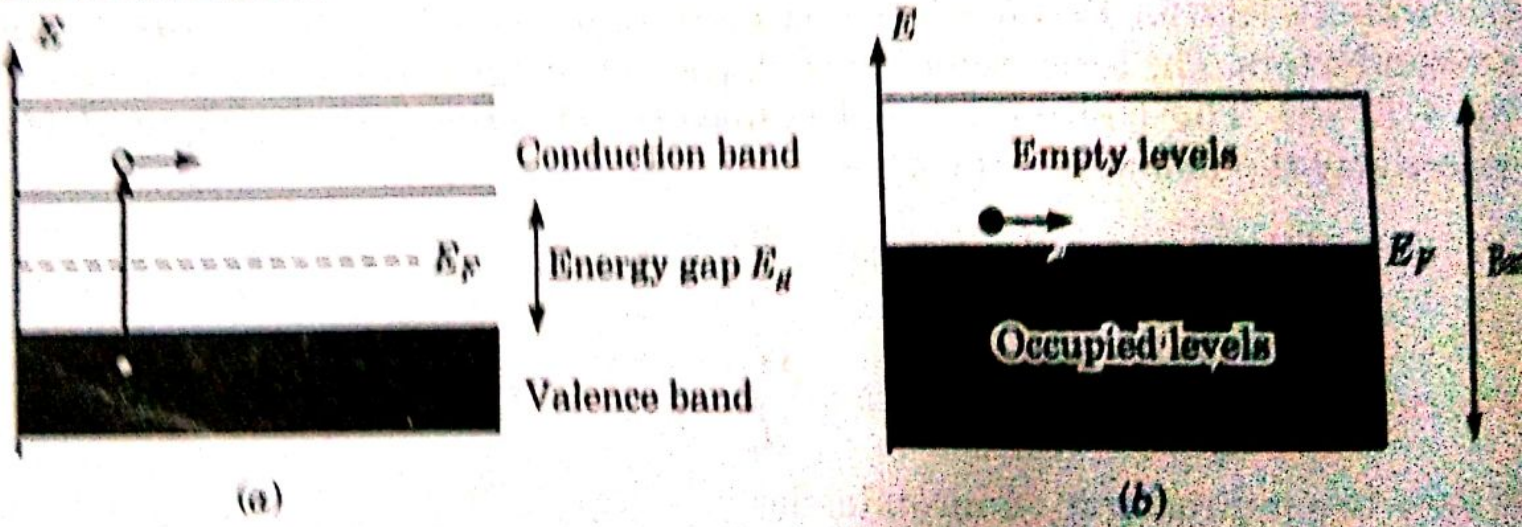

Fig. 4.14. Band in (a) Semiconductor (b) metals.

Insulators differ from semiconductors in that their energy gap is much larger than $K_B T$, where $K_B = (1.38066 \times 10^{-23}\ JK^{-1} = 0.08625\ meV.\ K^{-1})$ is the Boltzmann constant and T is the temperature in degrees K.

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